DETERMINATION OF CREDIT RISK BY THE USE OF CREDITRISK+ MODEL

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INTRODUCTION

CreditRisk+ is the method for the calculating the distribution of potential credit losses of the portfolio which was developed and published by Credit Suisse in 1997. This method can be used to determining of credit risk in retail and also in corporate sector, it means loans, derivatives and also marketable bonds. This method is based on portfolio approach to modelling risk of default, it is considering information relating to size and maturity of the instrument, credit quality (credibility) and the systematic risk of the borrower (the systematic risk is the risk arising from overall economic development affecting all subjects).

CreditRisk+ has one special problem regarding the aggregation of portfolio risk. It is the only model whose authors intend to avoid computer simulations to calculate portfolio risk and attain an analytical solution for the portfolio loss distribution. For this reason, the authors choose a Poisson approximation of the distribution of the number of defaulting credits in a portfolio segment. As a consequence each segment contains an infinitive number of credits [1].

This hidden assumption may lead to a significant overestimation of risk in small segments, e.g. when the segment of very large exposures in a bank portfolio is considered that is usually quite small. Thus, CreditRisk+ is particularly suited for very large and homogenous portfolios. However, at high percentiles, the reported portfolio losses even always exceed the total portfolio exposure [6].

In CreditRisk+ systematic risk factors are modelled as hidden variables that induce clients’s default probabilities to be gamma distributed with a given mean and variance. In order to be able to compute the portfolio loss distribution analytically, the authors of the model assume that systematic risk factors only refer to the clients in a specific sector while risk factors of different sectors are independent by supposition. Note that this presumption implies that clients in different sectors are independent as well, a problematic concealed structural decision for a portfolio model. Only clients who are at least partially represented in the same industrial sectors appear to be dependent in their default behaviour [8].

CreditRisk+ is statistical model of credit risk of default which does not create assumptions about causes of default. This approach is similar as approach which is used to market risk management (there is no effort to modelling of causes of changes in market prices. CreditRisk+ assumes that defaults are occurred in sequence of events so that cannot be predict exact time of their appearance and or their number.

To model the randomness of borrower default in the model are used mathematical methods that are often used in insurance industry. We expect portfolio with many individual risk with low probability of occurrence. It means that CreditRisk+ is analytical model which allowing quick and explicit calculation of total portfolio loss distribution. Model is based on value of property and bonds is essentially prospective and is determined by expected future of individual debtor from the perspective of investors. It connects actual credibility of debtors and their expected future development. Therefore it can be assumed that default rate varies continuously. Model CreditRisk+ considers default rate as continuous random variables. Standard deviations can be significant compared with default rates, it reflects real fluctuation of economic cycles. In practise we do not have individual default rates of individual debtors, appropriate method for determine default rates is for example assignment of default probability according to credit ratings.

External factors for example state of the economy can caused correlation between individual defaults although there is no causal relationship between them. Effects of these factors are processed into the model CreditRisk+ using the volatilities of default rates and analysis of sectors instead of using correlation of default as a direct input into the model [3].

Tab. 1: Comparison of some current model of measurement credit risk

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1 BASIC MODEL

Every model of calculating credit risk is dependent on input data which quality directly affects the accuracy of result. Model CreditRisk\textsuperscript{*} requires following input data [3]:

- exposure,
- default rates of individual debtors,
- volatility of individual default rates,
- default rate of return.

It is assumed:

- for a loan, the probability of default the term is the same for any equally time horizon,
- for large number of debtors, the probability of default of individual debtor is low and number of default which are appeared in a given time period is independent on amount of default in the past time horizons.

Under these assumptions, the probability distribution of the number of default during given time period well represented by a Poisson distribution with parameter $\mu$:

$$P(n) = \frac{\mu^n e^{-\mu}}{n!}, \text{ pre } n = 0,1,2,...,$$

(1)

where:

- $n$ - number of default,
- $\mu$ - average number of default for one year period,
- $\mu = \sum_{A} P_A$ - where $P_A$ is probability of default of debtor $A$.

Obviously, in portfolio is usually final quantity of bonds, so Poisson distribution which specifies the probability of default for infinite amount of default, only by approximating the distribution of default. If number of debtors are large enough, the probability is negligible that the number of defaults exceed number of debtors. If we assume Poisson distribution of default number, we expect that standard deviation of default rate will be approximated by square root of the mean default rate. In fact, we can observe that Poisson distribution underestimates the probability of default for all rating grades. This is due to variability of intensity of default in the time which is modelled as a function of changing the selected risk factors. If the average number of default is stochastic nature and has Gamma distribution with parameters $\mu$ and $\sigma^2_\mu$, can be represented by Poisson distribution.

1.1 DISTRIBUTION OF PORTFOLIO LOSSES

Derivation of individual asset risk is based on the calculation of expected loss. To derive the probability of losses well diversified portfolio, the losses are divided into groups according to the size of losses. Each group contains debtors with the same credit risk and is considered as independent portfolio of bonds with following marking [4]:

- $A$ – debtor,
- $L_A$ – possible loss,
- $P_A$ – probability of debtor default (A),
- $\xi_A$ - expected loss,
- $V_j$ – possible loss in the group $j$,
- $\xi_j$ - expected loss in the group $j$.
• $\mu_j$ - expected number of default in the group $j$.

With portfolio of bonds each group of debtors can be worked as an unchanging portfolio. Possible loss of each debtor in the group $j$ can be obtained as $v_j = jL$.

Under the definition we get:

$$\mathcal{E}_j = v_j \mu_j$$  \hspace{1cm} (2)

$$\mathcal{E}_A = L_A \cdot P_A$$  \hspace{1cm} (3)

Mark $\mathcal{E}_A$ expected loss of debtor $A$, it means $\mathcal{E}_A = \frac{n_A \lambda_A}{L}$ then $\mathcal{E}_j$ is expected loss in horizon of one year in the group $j$ expressed by the amount of expected losses $\mathcal{E}_A$ of all debtors in the group $j$, it means $\mathcal{E}_j = \sum_{A \in \mathcal{A}_j} \mathcal{E}_A$.

Expected number of defaults in horizon of one year in the group $j$ is:

$$\mu_j = \frac{\mathcal{E}_j}{v_j} = \sum_{A \in \mathcal{A}_j} \frac{\mathcal{E}_A}{v_j} = \sum_{A \in \mathcal{A}_j} \frac{\mathcal{E}_A}{v_A}$$  \hspace{1cm} (4)

Derivation of distribution the probability of losses for the whole portfolio consists of several steps.

1. **Derivation of probabilistic generating function for every group $j$**

   $$G_j(z) = \sum_{n=0}^{\infty} P(loss = nL) z^n = \sum_{n=0}^{\infty} P(n \text{ defaults}) z^{v_j}$$  \hspace{1cm} (5)

   Because we assume that the number of default is governed by the Poisson distribution. We can derive:

   $$G_j(z) = \sum_{n=0}^{\infty} \frac{e^{-\mu_j} \mu_j^n}{n!} z^{v_j} = \exp \left( -\mu_j + \mu_j z^{v_j} \right)$$  \hspace{1cm} (6)

2. **Derivation of probabilistic generating function for all portfolio**

   Assuming independence of each group is probabilistic generating function for whole portfolio:

   $$G(z) = \prod_{j=1}^{m} \exp \left\{ -\mu_j + \mu_j z^{v_j} \right\} = \exp \left\{ -\sum_{j=1}^{m} \mu_j + \sum_{j=1}^{m} \mu_j z^{v_j} \right\}$$  \hspace{1cm} (7)

   Where denotes expected number of default for the whole portfolio $\mu = \sum_{j=1}^{m} \mu_j$.

3. **Derivation of losses probability for all portfolio**

   From probabilistic generating function for whole portfolio, we can derive distribution probability of losses as:

   $$P(loss \text{ from } nL) = \frac{1}{n!} \left. d^n G(z) \right|_{z=0} \text{ for } n = 1, 2, ...$$  \hspace{1cm} (8)

**CONCLUSION**

Model CreditRisk+ is simple and easy implement model for calculation of expected losses in a state of default. CreditRisk+ is suitable model for calculate of credit risk of homogeneous portfolio consisting of a large number of debtors with low probability of default. It is based on Poisson approximation of individual default. Disadvantage of this model is that it does not involve the risk of downgrade. The model is in contrast to the method CreditMetrics model aims to determine of volume of venture capital assets, estimated distribution of expected losses and values in risk. Unlike KMV model, this method does not concentrate on relative risk of default to capital structure of the company. The model does not use Monte Carlo simulation therefore outputs are fully conditioned to input data. Probabilistic distribution of portfolio losses can be derived from probabilistic generating function with numerically stable algorithm.

And advantage of CreditRisk+ is that it requires a limited amount of data as inputs (basically only individual exposures and default probabilities), and the computation of the loan loss is rather easy to
perform. A limitation of the model is that a lot of ambiguity surrounds the specification of the default rates for individual obligors, which are actually basic inputs of the method. In CreditRisk⁺, obligors are not assigned to rating classes, and their characteristics do not determine these default rates. It is implicitly assumed that banks know these probabilities and their volatilities, but a concrete method to derive them is not offered. Another limitation is that the model does not assume market risks [5].

There are also some limitations to CreditRisk⁺. On a finer scale than default or survival, a change in the credit quality of an obligor that is captured as a transition of its internal or external rating is not reflected. Further, we mention the deterministic description of recoveries and the fact that large loss probabilities may lead to a distortion of the loss distribution due to multiple defaults arising from the Poisson approximation. On the other hand, however, more sophisticated models typically require more statistical input information, which in practice is often hard to identify.

REFERENCES


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DETERMINATION OF CREDIT RISK BY THE USE OF CREDISTRISK+ MODEL

Abstract
Risk express uncertainty associated with expected yield. Credit risk makes, that the issuer of bond may be not able to repay his debt and interests. It express credibility, reliability, the ability of issuers of securities to meet their commitments. Nowadays has become the issue of credit risk an important part of the life of each company, which has some claims against other institutions. Companies should not only measure credit risk but also try to calculate and predict potential default of the company in the future. This article deals with model CreditRisk+, which is based on typical insurance mathematics approach and therefore, also often called an acturial model. CreditRisk+ have become influential benchmarks for internal credit risk models. Practitioners and policy makers have invested in implementing and exploring each of the models individually, but have made less progress with comparative analyses.

Key words
CreditRisk+, calculation, portfolio, loss.

JEL Classification
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